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Stability of Snow Slabs

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Translated by Angie Thacker

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STABILITY OF SNOW SLABS

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BSTRACT

The comparison of stresses in the weakest layer with the corresponding criti-al curve in the mohr plane leads to a definition of primary stability. Investi-ation of this definition as a function of slope angle and distributed load, hows it to be in certain cases, independent of load.

Secondary stability characterizes the absence of sensitivity to perturba-ions, especially to a localized rupture of the critical layer. Here, considera-ion is given to different shapes of the fracture zone and also the influence of vertical concentrated force applied to the surface.

. INTRODUCTION

The rupture of snow slabs seems to be a principal cause of avalanche re-ease and may occur any time during the winter season so long as the conditions f snow, temperature, wind, and topography are favorable. When the temperature s sufficiently high, avalanches may release alternatively in a pear-shaped ashion; however, in most cases this does not shift large masses. It is there-ore of particular interest to study the stability of slabs.

A slab is in equilibrium when its weight is balanced by the forces which ct around its top, side, bottom and base. The first question that comes to ind is the relative importance of the basal forces in relation to the non-basal orces. This depends first of all on the ratio between the dimensions of the slab arallel to the slope and its thickness. No detailed statistics exist on this atio, however some indication may be deduced from the 1959-1963 reports of the

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wiss Federal Institute. The histogram of Fig. 1 shows that the distribution of he width of snow slabs released in the region of the Parsenn has a net maximum be- ween 20 and 40 m. The second maximum for the slabs larger than 100 meters is robably not realistic since these large slabs are often composed of several maller slabs. Slab thickness is generally unknown, but it may be estimated to verage about 0.5 meters. Thus the ratio of width to thickness usually exceeds 0. In ten cases where the stratigraphic profile of the slab was actualiy ex- mined, this ratio was always higher than 50. In addition, the relative mportance of the basal force depends on the snow strength at the base in com- arison to snow strength throughout the thickness of the slab; here also, umerical data is rare. Rammsonde measurements in certain instances show ratios f up to 5 to 1 between the strength of the base and the slab profile.

Thus it seems that in the majority of cases basal conditions determine tability, the periphery only intervening when a critical state exists at the ase or in certain cases conditioned by the terrain (couloirs). Therefore the ollowing analysis will be limited to slabs which extend infinitely in two imensions and fail only along the basal plane.

. STATIC CONDITIONS

Let us consider an infinite slab of thickness, h, sloped at an angle, ψ. e adopt a system of rectangular coordinates with axes x, y, and z (see Fig.2). his is the case of plane stress studied by Haefeli (1942) and is defined in art by the following equations

$$\frac{\partial \sigma_y}{\partial y} = \gamma \cos\psi, \quad \frac{\partial \sigma_x}{\partial x} = 0, \quad \frac{\partial \tau_{xy}}{\partial y} = \gamma \sin\psi \qquad (1)$$

here γ is the density of the snow.

Assume that the velocity components, V_x, V_y, and V_z depend on stress accord- ng to an extension of the plasticity equations of Lame

$$\frac{\partial V_x}{\partial x} = a\left[\sigma_x - \frac{1}{m}(\sigma_y + \sigma_z)\right]$$

$$\frac{\partial V_x}{\partial y} + \frac{\partial V_y}{\partial x} = 2a\left(1 + \frac{1}{m}\right)\tau_{xy} \qquad (2)$$

th permutations of the indexes for the other components. In Equation (2), is the plastic poisson number and a represents a coefficient of viscosity. is obvious that this representation can give only a first approximation nce there is no guarantee that stress is a linear function of velocity. so, m does not have the same value in tension as it has in compression. us we must be careful not to attribute too much precision to the ensuing alysis.

For the case of plane stress on an infinitely extended slope, the velity gradients in the x and z directions vanish

$$\frac{\partial V_x}{\partial x} = 0 \quad ; \quad \sigma_x - \frac{1}{m}(\sigma_y + \sigma_z) = 0$$

$$\frac{\partial V_z}{\partial z} = 0 \quad ; \quad \sigma_z - \frac{1}{m}(\sigma_x + \sigma_y) = 0 \qquad (3)$$

us the following expressions are obtained

$$\sigma_y = \cos\psi \int_0^y \gamma\, dy \quad , \quad \tau_{xy} = \sin\psi \int_0^y \gamma\, dy$$

$$\sigma_x = \sigma_z = \frac{\sigma_y}{m-1} = \frac{\cos\psi}{m-1} \int_0^y \gamma\, dy \qquad (4)$$

e local angle of creep is defined as

$$\tan\beta = \frac{\partial V_y}{\partial y} \Big/ \frac{\partial V_x}{\partial y} \qquad (5)$$

cause the derivatives along x disappear, this is equal to

$$\frac{m-2}{2(m-1)} \cot\psi \qquad (6)$$

tersects the τ - axis at τ_1 , the criteria of primary stability is simply

$$\tau_1 < \tau_c \tag{10}$$

, is found from

$$\tau_1 = \left[\frac{(M^2 + \tan^2\psi)^{1/2}}{\cos\omega} - (1-M)\tan\psi \right] g\cos\psi \tag{11}$$

ere M, the "compressibility," is defined as

$$M = \frac{m-2}{2m-2}$$

$$0 \leq M \leq 1/2 \tag{12}$$

t us first consider homogeneous slabs of equal thickness and examine the varia-
on of stability with slope angle. In Fig. 4, τ_1/g is shown as a function of
' for different values of the parameter ω , and for two extreme values of

$$m = \infty \quad , \quad m = 2$$

can be observed that for slabs of equal thickness, steeper slopes are more criti-
l. However, let us next consider the case of slab thickness varying with the
ope angle. This situation is observed when snow falls with little wind. Then,
must be replaced by $g_0 \cos\psi$, where g_0 is the slab thickness on level ter-
in. As shown in Fig. 5, the shape of the curves change. τ_1 is maximum be-
een 35° and 50° for $m = \infty$; and is maximum between 45° and 65° for $m = 2$.
s indicates that if τ_1 increases progressively up to τ_c (a continuous fall
snow), slopes of about 45° rupture before the steeper slopes.

If only slopes of practical interest are considered, say below 50°, primary
bility is assured for all slopes if the load, g , is less than 80% of τ_c
% with constant snow height).

Since stability varies as g changes, it is possible to define a limiting
ope which is "absolutely stable" independent of an increase in g . In fact,
can be seen in Fig. 4 and Fig. 5 that if m is small enough, τ_1 can be less

an zero. Since $\tau_c > 0$, Equation 10 is satisfied under this condition for all lues of g . The slope angle, ψ_1 , of "primary absolute stability" is found setting $\tau_1 = 0$ in Equation (10)

$$\tan^2 \psi_1 = (1-M)^2 \sin^2 \omega - M^2 \qquad (13)$$

plot of Equation (13) is shown in Fig. 6. ψ_1 exists only if M is suffiently small, i.e. if the snow is sufficiently incompressible, and increases a uniform manner with the parameter, ω .

If stability is not initially absolute, the situation may improve accordg to particular conditions. A settling of the snow causes a decrease in m d consequently a decrease in M . According to Fig. 7, this results in a crease in τ_1 and therefore an increase in stability. This effect becomes eater as the slope angle decreases and ω increases.

SECONDARY STABILITY

Secondary stability is only assured for a slab which has primary stability d which remains stable if it is submitted to a perturbation. The perturban may be, for example, a local break in the critical layer. The forces susned by the critical layer are transferred to neighboring zones with a marked ximum along the boundaries of the slab and a possible propagation of the fracre.

Two extreme cases are observed for the configuration of fracture lines: ircular pattern and an extended horizontal pattern. The first case is analos to considering the stress on a cylinder of radius, r_0 , and thickness . The exact solution of this problem is very complicated and depends on the co-elastic properties of all the layers in the slab.

We limit our analysis to the simple case which rests on the following hyposes. First, the component of the weight of the cylinder which is normal to

e slope is always balanced (even after failure) by the supporting base, only e tangential components are redistributed along the boundary; and second, e stresses created by a concentrated force are propagated through the snow cording to an attenuation function which can be expressed as a power series $\frac{1}{r}$, where r is the distance to the point of application of the concen- ated force. If r is relatively large, the term of the series with the allest exponent, k, is the dominating term and the other terms can be dis- garded. Then, the increase in stress caused by a concentrated force is

$$\Delta\tau = \frac{T(k-2.)}{2\pi r_0^2}\left(\frac{r_0}{r}\right)^k \tag{14}$$

d according to the first hypothesis

$$T = g\pi r_0^2 \sin\psi = \pi r_0^2 \tau \tag{15}$$

e stress change at $r = r_0$ becomes

$$\Delta\tau_{r=r_0} = \tau\left(\frac{k}{2} - 1\right) \tag{16}$$

d the total stress at $r = r_0$ is now

$$\tau + \Delta\tau = \frac{k}{2}\tau \tag{17}$$

e exponent, k is determined by the viscosity of the layer. When the vis- sity is large, the forces are more concentrated, and therefore k is large; en the viscosity is small, the forces are distributed to a great distance and approaches 2 in the limit. This suggests the comparison

$$m = \infty \;,\; k = \infty$$

$$m = 2 \;,\; k = 2 \tag{18}$$

d further suggests as a first approximation

$$k \simeq m \tag{19}$$

is then possible to replace τ by

$$\tau\left(\frac{1-M}{1-2M}\right) \tag{20}$$

With reference to the ψ_1 of Equation (11), it is possible to define the iting angle of incline, ψ_2' , for the "secondary absolute stability" of a pe as

$$\psi_2' = \frac{1-2M}{1-M}\tan\psi_1 \tag{21}$$

the slope angle, ψ is less than ψ_2' , then the slope remains stable in te of an arbitrarily large radius of fracture in the critical layer.

The $1/r$ dependence given by Equation (14) suggests that the increase in ess is finite even if the radius of the fracture line becomes infinite. wever, if stress is propagated according to an exponential law, then the ess may diverge. This problem has not been solved for snow, although sinesq (see Kollbrunner) has worked on a similar theory for soils.

When the radius of the fracture line is small, the increase in stress is er than the value given by Equation (16). It varies at first as r_o^2 and n approaches assymptotically a radius of the same order of magnitude as depth .

The second extreme case, an extended horizontal fracture line, can be repre- ted by a band of width $2X_o$ in the x-direction and infinitely long in the irection (see Fig. 2). For this case the increase in stress is

$$\Delta\tau = \frac{T'(K-2)}{X_o}\left(\frac{X_o}{x}\right)^{K-1} \tag{22}$$

re

$$T' = g X_0 \sin\psi = \tau X_0 \tag{23}$$

change in stress at $X = X_0$ is

$$\Delta\tau = \tau(K-2) \tag{24}$$

the total stress at $X = X_0$ is

$$\frac{\tau}{1-2M} \tag{25}$$

this case, the steepest angle, ψ_2'' of "secondary absolute stability" is

en by

$$\tan(\psi_2'')^2 = (1-2M)\tan\psi_1 \tag{26}$$

ψ_2'' is slightly smaller than ψ_2' because the boundaries to which the stress transferred is larger for the case of the cylinder.

In Fig. 8 the angles ψ_1, ψ_2' and ψ_2'' are shown as functions of M for ee values of the parameter ω .

The importance of the "compressibility factor," M , can be observed. During tling and destructive metamorphism, M decreases, consequently, stability in- ases. However, if the critical layer is near the ground, constructive meta- phism tends to increase M .

As mentioned at the beginning of this section, primary stability is a pre- uisite for secondary stability: if the critical curve is reached, a local cture can only increase the stress at the boundaries, amplify the instability, cause the fracture to be propagated ad infinitum.

STABILITY WITH A CONCENTRATED FORCE ACTING ON THE SURFACE

Vertical forces may be applied to a slab by the pass of a skier. The esses are distributed throughout the snow with a maximum stress approxi- ely below the point of application. As with the original load, the vertical ce can be resolved into a tangential and normal component. Assuming that the

tical layer is intact at the moment of application of the force, then, if the le of the slope is less than ψ_1 , the concentrated force can be increased itrarily without leading to failure.

It is convenient to introduce a dimensionless force, $\varkappa$, defined by

$$\varkappa = \frac{\chi}{\pi y^2 g} \tag{27}$$

re χ is the applied force and $\pi y^2 g$ represents the weight of a cylinder whose ght and radius are equal to the depth, y . Since τ_1 increases linearly with (Fig. 9), there is a value $\varkappa_c$ which corresponds to τ_c (Point A). For a cific radius of fracture, r_0 , a curve can be constructed which passes ough Point A. Consequently, the critical layer fails and the fracture is pagated up to a radius, r_{01} . For a very large r_0 , the influence of K appears in relation to the weight of the cylinder of snow. Graphically, s is represented by a horizontal line which intersects the former straight e at B with abcissa $\varkappa_B$ and ordinate $\tau_{1\infty}$. If τ_c is greater than $\tau_{1\infty}$, χ is large enough to initiate a fracture, then A is to the right of B and fracture propagates. The abcissa of B, $\varkappa_B$, is thus the maximum force t can be applied without inducing fracture propagation. As shown in Fig. 10, is a function of the slope angle ψ , the compressibility M , and the para- er w .

The angle ψ_1, exists for $0 < M < M_1$

$$M_1 = \frac{\sin w}{(1+\sin w)} \tag{28}$$

$\psi < \psi_1$, no fracture can be produced and hence $\varkappa_B$ does not exist; however, $\psi > \psi_1$, $\varkappa_B$ exists and is greater than unity. For $M_1 < M < \frac{1}{2}$, $\varkappa_B$ always sts and is less than unity for slopes less than 20°. Thus, χ is approximately g for absolute stability. If the horizontal fracture line is considered, as example produced by an elongated ski track, a similar analysis can be made,

ever, as a result of Equation (21) and (26), $\tan\psi$ must be replaced by $\frac{\tan\psi}{1-M}$.

LUSIONS

It seems that this study describes rather well the observed phenomona; how- quantitative values cannot be attributed because the hypotheses which were for their analytical simplicity are only approximations. It is likely that introduction of the number m, and the derived M which describe the plastic vior of the snow, does not stray too far from reality. On the other hand, hypotheses concerning the propagation of stress in the section on secondary ility should be received with more skepticism,- results from these hypo- es may entail a large factor of error. It must also be realized that it is icult to procure experimental values of τ_c, w, m etc. Measurements in the upper or lower portions of an avalanche zone do not necessarily ribe the mean values in the critical layers of the slab prior to release. rder to compare theory with experiment, it is necessary to measure and aver- the parameters of the very slab which is about to release! The slab would have to represent the theoretical conditions that were chosen in this study: e, constant slope, and free from natural anchors. In view of these diffi- ies, it seems that this study, inexact as it is, may already exceed what can etermined by experiment.

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2 Reference of coordinate axes, stresses and angles.

3 Mohr's plane, principal stresses and definition of τ_c .

4 τ_1/g as a function of the angle of the slope for constant snow thickness.

5 τ_1/g_0 as a function of the angle of the slope for constant snow height.

6 Maximum slope for primary absolute stability.

7 $\tau_1/g \cos\psi$ as a function of the compressibility, M.

8 Maximum slope for secondary absolute stability.

9 Fracture line radius for a concentrated force.

10 Maximum concentrated force as a function of the slope angle for different values of the compressibility, M, at $\omega = 30^\circ$.

Fig.1

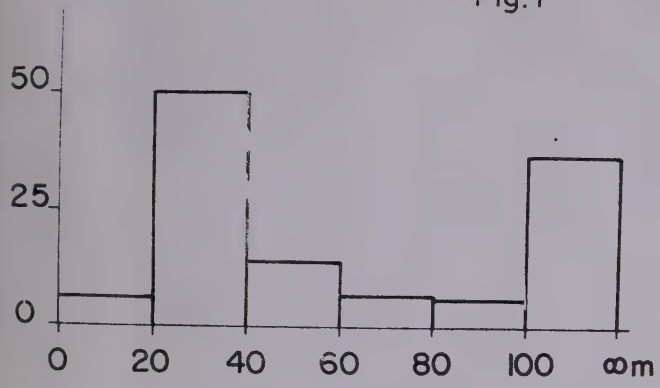


Fig. 2

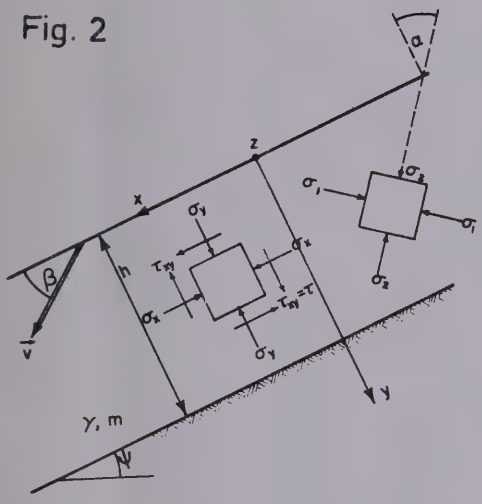


Fig. 3

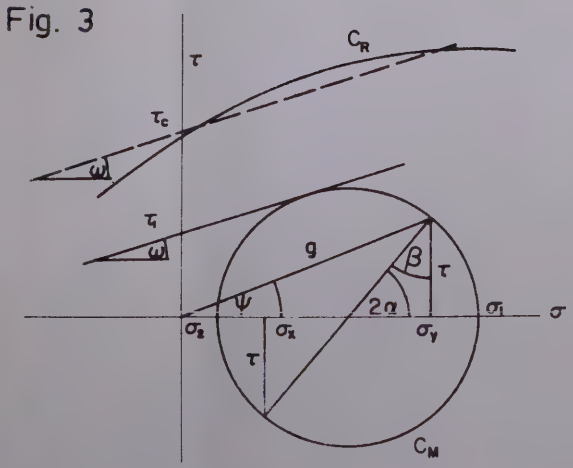


Fig.4

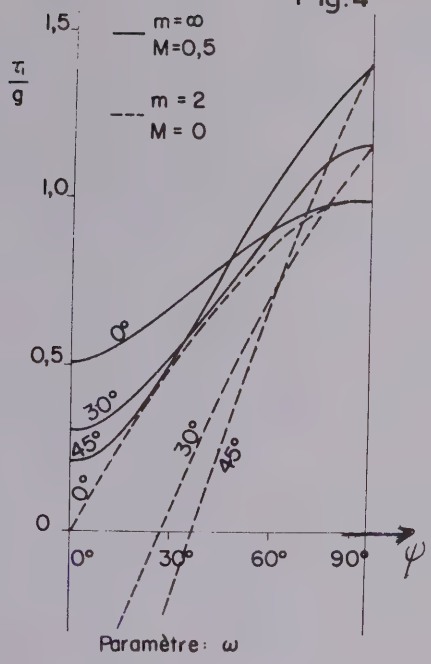


Fig. 5

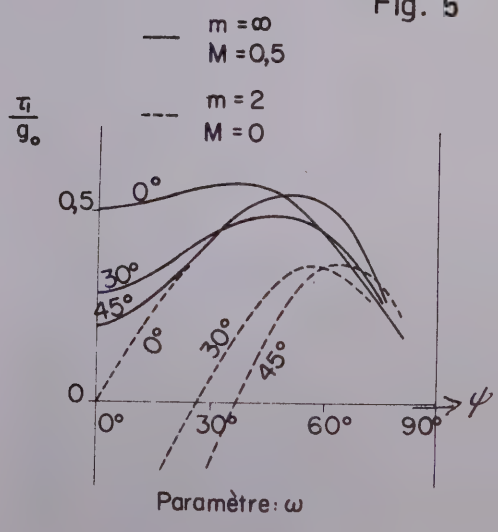


Fig. 6

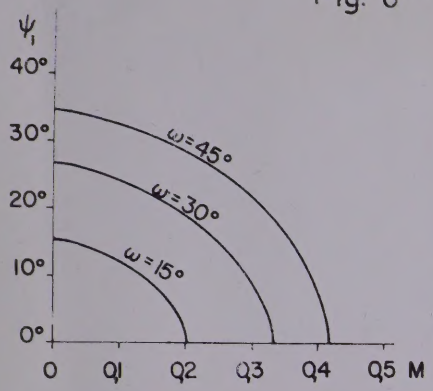


Fig. 9

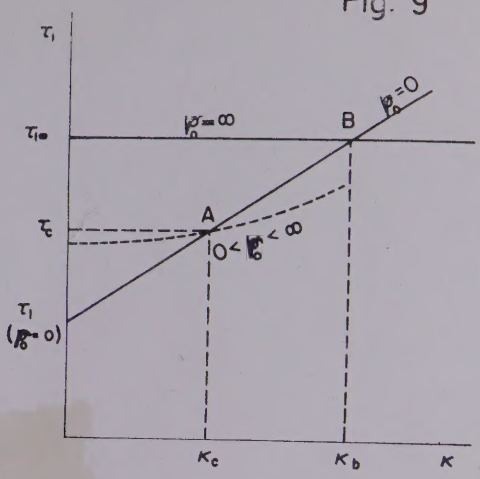


Fig. 7

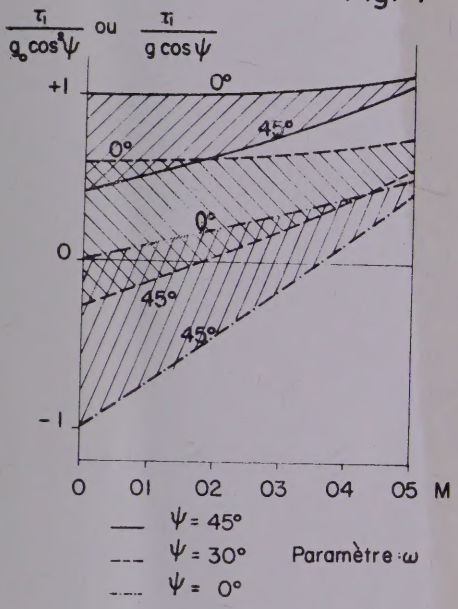


Fig. 10

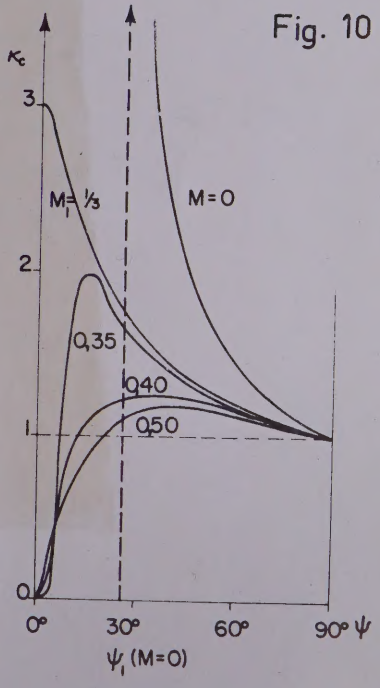
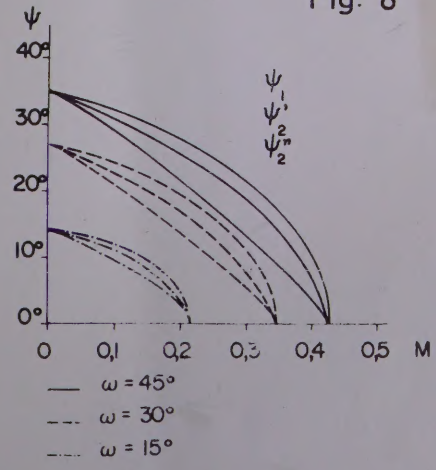


Fig. 8



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